

# Honors Algebra II

## Notes Section 8.6

### Translate and Classify Conic Sections

#### Standard Form of Equations of Translated Conics

In the following equations, the point  $(h, k)$  is the *vertex* of the parabola and the *center* of the other conics.

Circle  $(x - h)^2 + (y - k)^2 = r^2$

Horizontal axis

Vertical axis

Parabola  $(y - k)^2 = 4p(x - h)$

$(x - h)^2 = 4p(y - k)$

Ellipse  $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$

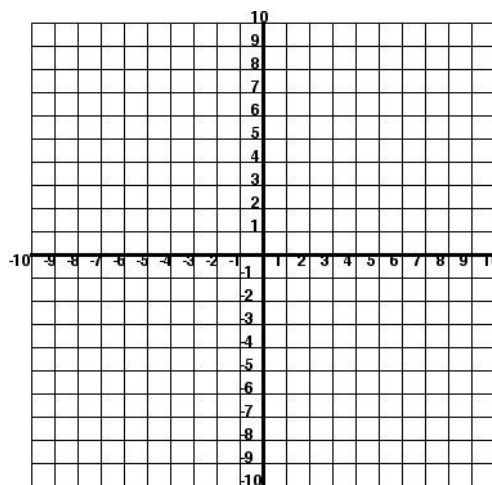
Hyperbola  $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$

$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$

**EXAMPLE 1** Graph.  $(x - 2)^2 + (y + 3)^2 = 9$ .

**STEP 1** Identify the radius.

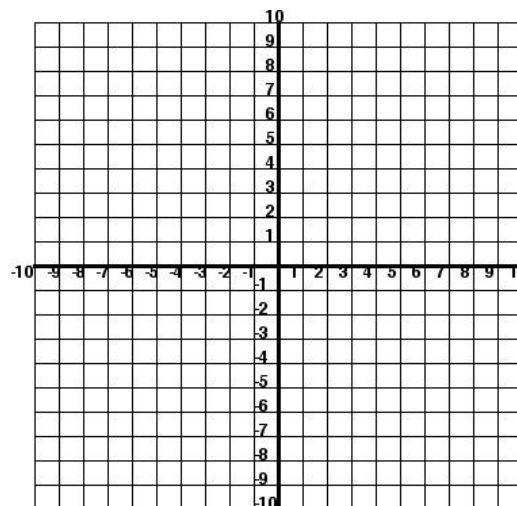
**STEP 2** Graph the, plot the points to form the radii and complete the circle.



**EXAMPLE 2a**  $(x + 3)^2 - \frac{(y - 4)^2}{4} = 1$       **STEP 1** Vertical or Horizontal.

**STEP 2** Identify a, b and c.

**STEP 3** Find the Center, Vertices, Co-Vertices, Foci, and Asymptotes.



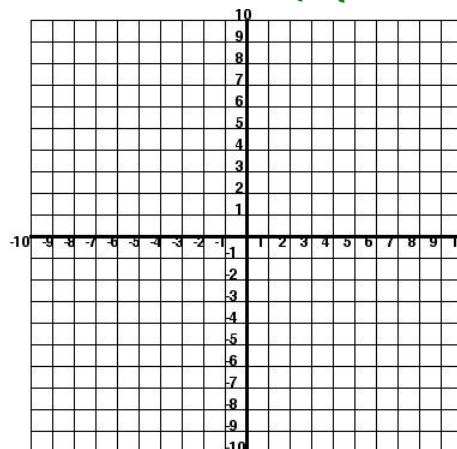
**STEP 4** Graph

b) Graph  $\frac{(y - 3)^2}{4} - \frac{(x + 1)^2}{9} = 1$ .

**STEP 1** Vertical or Horizontal.

**STEP 2** Identify a, b and c.

**STEP 3** Find the Center, Vertices, Co-Vertices, Foci, and Asymptotes.



**STEP 4** Graph

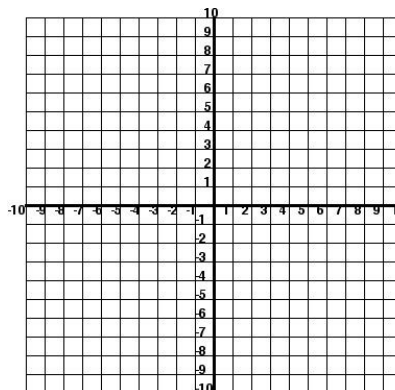
**EXAMPLE 3** Write an equation of the parabola whose vertex is at  $(-2, 3)$  and whose focus is at  $(-4, 3)$ .

**STEP 1** Sketch to determine the form of the parabola.

**STEP 2** Identify  $h$  and  $k$ .

**STEP 3** Find  $p$ .

**STEP 4** Write equation.

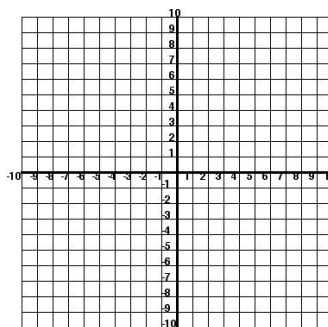
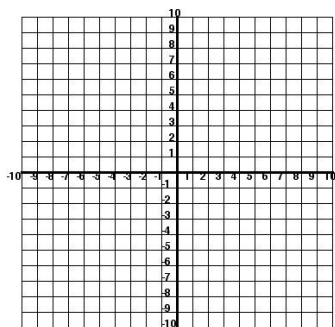


**EXAMPLE 3** Write an equation of the ellipse with the given foci and co-vertices.

a) Foci:  $(1, 2)$  and  $(7, 2)$   
 Co-V:  $(4, 0)$  and  $(4, 4)$

b) Foci:  $(3, 5)$  and  $(3, -1)$   
 Co-V:  $(1, 2)$  and  $(5, 2)$

**STEP 1** Sketch to determine the form of the ellipse.



**STEP 2** Find the center and identify  $h$  and  $k$ .

**STEP 3** Find  $b$  and  $c$ .

**STEP 4** Find  $a$ .

**STEP 5** Write the equation.

**EXAMPLE 5** Identify the line(s) of symmetry for each conic section in EXAMPLES 1-4.

a) **EXAMPLE 1** \_\_\_\_\_

b) **EXAMPLE 2a** \_\_\_\_\_

**EXAMPLE 2b** \_\_\_\_\_

c) **EXAMPLE 3** \_\_\_\_\_

d) **EXAMPLE 4a** \_\_\_\_\_

**EXAMPLE 4b** \_\_\_\_\_

**Classifying Conics Using Their Equations**

Any conic can be described by a **general second-degree equation** in  $x$  and  $y$ :  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ . The expression  $B^2 - 4AC$  is the **discriminant** of the equation and can be used to identify the type of conic.

Discriminant	Type of Conic
$B^2 - 4AC < 0, B = 0, \text{ and } A = C$	Circle
$B^2 - 4AC < 0$ and either $B \neq 0$ or $A \neq C$	Ellipse
$B^2 - 4AC = 0$	Parabola
$B^2 - 4AC > 0$	Hyperbola

If  $B = 0$ , each axis of the conic is horizontal or vertical.

**EXAMPLE 6** Classify the conic section given. Then graph their equation.

a)  $4x^2 + y^2 - 8x - 8 = 0$

a = \_\_\_\_\_ b = \_\_\_\_\_ c = \_\_\_\_\_

Center: \_\_\_\_\_

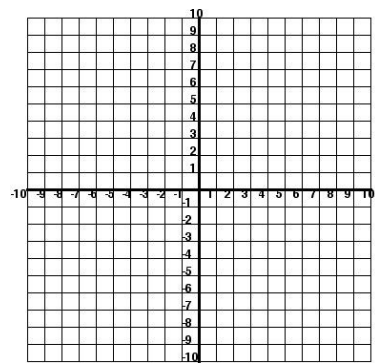
a = \_\_\_\_\_

b = \_\_\_\_\_ c = \_\_\_\_\_

Vertices: \_\_\_\_\_

Co-Vertices: \_\_\_\_\_

Foci: \_\_\_\_\_



b)  $x - 3 = 1/2(y-2)^2$

a = \_\_\_\_\_ b = \_\_\_\_\_ c = \_\_\_\_\_

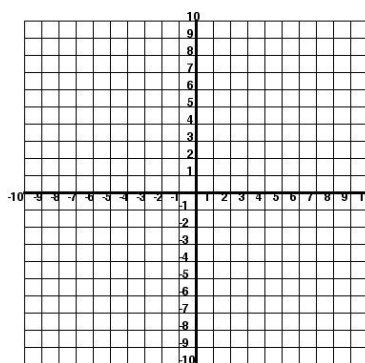
Vertex: \_\_\_\_\_

4p = \_\_\_\_\_

p = \_\_\_\_\_

Focus: \_\_\_\_\_

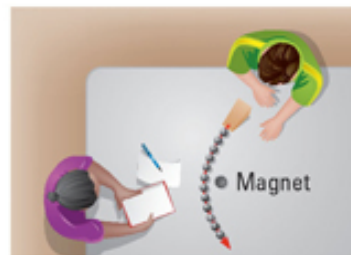
Directrix: \_\_\_\_\_



**EXAMPLE 7** In a lab experiment, you record images of a steel ball rolling past a magnet. The equation  $16x^2 - 9y^2 - 96x + 36y - 36 = 0$ . Models the ball's path.

a) What is the shape of the path?

b) Write an equation for the path in standard form.



$$16x^2 - 9y^2 - 96x + 36y - 36 = 0$$

c) Graph the equation.

a = \_\_\_\_\_

b = \_\_\_\_\_

c = \_\_\_\_\_

Center: \_\_\_\_\_

Vertices: \_\_\_\_\_

Co-Vertices: \_\_\_\_\_

Foci: \_\_\_\_\_

